# CS 61A Fall 2024

# Tree Recursion

Discussion 4: September 25, 2024

If you do not have an in-person TA, you can reach your TA using this Zoom link.

If there are fewer than 3 people in your group, feel free to merge your group with another group in the room.

Now switch to Pensieve:

- **Everyone**: Go to pensieve.co, log in with your @berkeley.edu email, and **enter your group number** (which was in the email that assigned you to this lab).

Once you're on Pensieve, you don't need to return to this page; Pensieve has all the same content (but more features). If for some reason Penseive doesn't work, return to this page and continue with the discussion.

## Getting Started

Say your name and your favorite tree (a particular tree or a kind of tree) in honor of today's topic: tree recursion.

**Definition**: Tree recursive functions are functions that call themselves more than once.

In this discussion, don't use a Python interpreter to run code until you are confident your solution is correct. Figure things out and check your work by *thinking* about what your code will do. Not sure? Talk to your group!

[**New**] Recursion takes practice. Please don't get discouraged if you're struggling to write recursive functions. Instead, every time you do solve one (even with help or in a group), make note of what you had to realize to make progress. Students improve through practice and reflection.

[**For Fun**] This emoticon of a guy in a cowboy hat is valid Python: `o[:-D]`

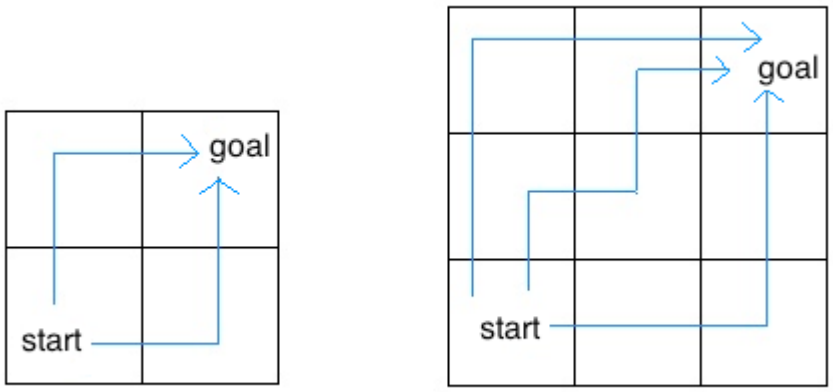
```
>>> o = [2, 0, 2, 4]
>>> [ o[:-D] for D in range(1,4) ]
[[2, 0, 2], [2, 0], [2]]
```

## Tree Recursion

For the following questions, don't start trying to write code right away. Instead, start by describing the recursive case in words. Some examples: - In `fib` from lecture, the recursive case is to add together the previous two Fibonacci numbers. - In `double_eights` from lab, the recursive case is to check for double eights in the rest of the number. - In `count_partitions` from lecture, the recursive case is to partition `n-m` using parts up to size `m` **and** to partition `n` using parts up to size `m-1`.

### Q1: Insect Combinatorics

An insect is inside an `m` by `n` grid. The insect starts at the bottom-left corner `(1, 1)` and wants to end up at the top-right corner `(m, n)`. The insect can only move up or to the right. Write a function `paths` that takes the height and width of a grid and returns the number of paths the insect can take from the start to the end. (There is a closed-form solution to this problem, but try to answer it with recursion.)




**Insect grids.**

In the `2` by `2` grid, the insect has two paths from the start to the end. In the `3` by `3` grid, the insect has six paths (only three are shown above).

**Hint:** What happens if the insect hits the upper or rightmost edge of the grid?

```
def paths(m, n):
    """Return the number of paths from one corner of an
    M by N grid to the opposite corner.

    >>> paths(2, 2)
    2
    >>> paths(5, 7)
    210
    >>> paths(117, 1)
    1
    >>> paths(1, 157)
    1
    """
    "*** YOUR CODE HERE ***"
```

**Presentation Time:** Once your group has converged on a solution, it's time to practice your ability to describe why your recursive case is correct. Nominate someone and have them present to the group for practice. Then, tell this description to your TA for feedback (on Zoom if your TA is remote).

# Tree Recursion with Lists

[**New**] Some of you already know list operations that we haven't covered yet, such as `append`. Don't use those today. All you need are list literals (e.g., `[1, 2, 3]`), item selection (e.g., `s[0]`), list addition (e.g., `[1] + [2, 3]`), `len` (e.g., `len(s)`), and slicing (e.g., `s[1:]`). Use those! There will be plenty of time for other list operations when we introduce them next week.

The most important thing to remember about lists is that a non-empty list `s` can be split into its first element `s[0]` and the rest of the list `s[1:]`.

```
>>> s = [2, 3, 6, 4]
>>> s[0]
2
>>> s[1:]
[3, 6, 4]
```

### Q2: Max Product

Implement `max_product`, which takes a list of numbers and returns the maximum product that can be formed by multiplying together non-consecutive elements of the list. Assume that all numbers in the input list are greater than or equal to 1.

```
def max_product(s):
    """Return the maximum product of non-consecutive elements of s.

    >>> max_product([10, 3, 1, 9, 2])   # 10 * 9
    90
    >>> max_product([5, 10, 5, 10, 5])  # 5 * 5 * 5
    125
    >>> max_product([])                 # The product of no numbers is 1
    1
    """
```

First try multiplying the first element by the `max_product` of everything after the first two elements (skipping the second element because it is consecutive with the first), then try skipping the first element and finding the `max_product` of the rest. To find which of these options is better, use `max`.

A great way to get help is to talk to the course staff!

Complete this sentence together: "The recursive case is to choose the larger of … and …"

**Description Time:** Now try to complete this sentence together: "The recursive case is to choose the larger of _____ and _____." When you're done, see how your answer compares to ours.

The recursive case is to choose the larger of the largest product that includes the first but not the second element and the largest product that does not include the first element.

### Q3: Sum Fun

Implement `sums(n, m)`, which takes a total `n` and maximum `m`. It returns a list of all lists: 1. that sum to `n`, 2. that contain only positive numbers up to `m`, and 3. in which no two adjacent numbers are the same.

Two lists with the same numbers in a different order should both be returned.

Here's a recursive approach that matches the template below: build up the `result` list by building all lists that sum to `n` and start with `k`, for each `k` from 1 to `m`. For example, the result of `sums(5, 3)` is made up of three lists: - `[[1, 3, 1]]` starts with 1, - `[[2, 1, 2], [2, 3]]` start with 2, and - `[[3, 2]]` starts with 3.

**Hint:** Use `[k] + s` for a number `k` and list `s` to build a list that starts with `k` and then has all the elements of `s`.

```
>>> k = 2
>>> s = [4, 3, 1]
>>> [k] + s
[2, 4, 3, 1]
```

```
def sums(n, m):
    """Return lists that sum to n containing positive numbers up to m that
    have no adjacent repeats.

    >>> sums(5, 1)
    []
    >>> sums(5, 2)
    [[2, 1, 2]]
    >>> sums(5, 3)
    [[1, 3, 1], [2, 1, 2], [2, 3], [3, 2]]
    >>> sums(5, 5)
    [[1, 3, 1], [1, 4], [2, 1, 2], [2, 3], [3, 2], [4, 1], [5]]
    >>> sums(6, 3)
    [[1, 2, 1, 2], [1, 2, 3], [1, 3, 2], [2, 1, 2, 1], [2, 1, 3], [2, 3, 1], [3, 1, 2],
    [3, 2, 1]]
    """
    if n < 0:
        return []
    if n == 0:
        sums_to_zero = []     # The only way to sum to zero using positives
        return [sums_to_zero] # Return a list of all the ways to sum to zero
    result = []
    for k in range(1, m + 1):
        result = result + [ ___ for rest in ___ if rest == [] or ___ ]
    return result
```

`k` is the first number in a list that sums to `n`, and `rest` is the rest of that list, so build a list that sums to `n`.

Call `sums` to build all of the lists that sum to `n-k` so that they can be used to construct lists that sum to `n` by putting a `k` on the front.

Here is where you ensure that "no two adjacent numbers are the same." Since `k` will be the first number in the list you're building, it must not be equal to the first element of `rest` (which will be the second number in the list you're building).

If you get stuck (which many groups do), ask for help!

# Document the Occasion

Please all fill out the attendance form (one submission per person per week).